
**PUNCTE REMARCABILE
ÎN TRIUNGHI
DISTANȚE. INEGALITĂȚI
DE LA INIȚIERE LA PERFORMANȚĂ**

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Identități remarcabile într-un triunghi

„Fiecare om are un orizont. Când acest orizont se îngustează foarte mult, el devine un punct, iar omul exclamă: **Iată punctul meu de vedere.**”

David Hilbert

§ 1.1. Identități remarcabile cu laturi și raze

$$\sum (p-a)(a+b)(a+c) = p(p^2 + 5r^2 + 8Rr).$$

$$\sum a^2 = 2(p^2 - r^2 - 4Rr).$$

$$\sum a^2 = 2p(p^2 - 3r^2 - 6Rr).$$

$$\sum a^4 = 2[p^4 - p^2(8Rr + 6r^2) + r^2(4R+r)^2].$$

$$\sum b^2c^2 = p^4 + p^2(2r^2 - 8Rr) + r^2(4R+r)^2.$$

$$\sum a^2(p-a)^2 = 2r^2[(4R+r)^2 - p^2].$$

$$\sum \frac{a(b+c)^2}{bc(p-a)} = \frac{p^2 - 3r^2}{r^2}.$$

$$\sum \frac{(b+c)^2}{b^2c^2(p-a)} = \frac{p^2 - 3r^2}{4Rr^3p}.$$

$$\sum a^2(b+c)^2(p-b)(p-c) = 4p^2Rr(p^2 - 3r^2).$$

$$\sum \frac{a^2(b+c)^2}{p(p-a)} = \frac{4R}{r}(p^2 - 3r^2).$$

$$\sum a^2(b+c)(p-b)(p-c) = 2pr[p^2(2R-r) - r^2(4R+r)].$$

$$\sum \frac{a(b+c)^2}{p(p-a)} = \frac{2}{rp}[p^2(2R+r) + r^2(4R+r)].$$

$$\sum a(b+c)^2(p-b)(p-c) = 2rp[p^2(2R+r) + r^2(4R+r)].$$

$$\sum \frac{a^2(b+c)}{p(p-a)} = \frac{2}{rp}[p^2(2R-r) - r^2(4R+r)].$$

$$\sum \frac{a(p-b)(p-c)}{(a+b)^2(a+c)^2} = \frac{r[p^2(2R+r)+r^2(4R+r)]}{2p(p^2+r^2+2Rr)^2}.$$

$$\sum \frac{p-a}{b+c} = \frac{p^2+5r^2+8Rr}{2(p^2+r^2+2Rr)}. \quad \sum \frac{p-a}{a} = \frac{p^2+r^2-8Rr}{4Rr}.$$

$$\sum bc(b^2+c^2) = 2p^4 - 8p^2Rr - 2r^2(4R+r)^2.$$

$$\sum bc(b^3+c^3) = 2p[p^4 - p^2(2r^2+4Rr) - r^2(8R^2+14Rr+3r^2)].$$

$$\sum \frac{b^2+c^2}{p-a} = \frac{2p^2(2R+3r) - 2r(4R+r)^2}{rp}.$$

$$\sum (b^2+c^2)(p-b)(p-c) = 2p^2(2Rr+3r^2) - 2r^2(4R+r)^2.$$

$$\sum (b+c)(p-b)^3(p-c)^3 = 2r^3p[(4R+r)^2(2R+r) - p^2(6R+r)].$$

$$\sum \frac{(b+c)(p-b)(p-c)}{(p-a)^2} = \frac{2}{rp}[(4R+r)^2(2R+r) - p^2(6R+r)].$$

$$\sum bc(b+c)(p-b)(p-c) = 2r^2p[p^2 + (4R+r)(2R+r)].$$

$$\sum \frac{b+c}{a(p-a)} = \frac{p^2 + (4R+r)(2R+r)}{2Rrp}.$$

$$\sum a(b+c)(p-b)^2(p-c)^2 = 2r^2p^2(8R^2+4Rr-r^2-p^2).$$

$$\sum \frac{(b+c)(p-b)(p-c)}{bc(p-a)} = \frac{8R^2+4Rr-r^2-p^2}{2Rr}.$$

$$\sum \frac{(b+c)^2}{bc} = \frac{p^2+r^2+10Rr}{2Rr}. \quad \sum a(a+b)(a+c) = 4p(p^2-r^2-Rr).$$

$$\sum a^2(a+b)(a+c) = 4p^2(p^2-3r^2-4Rr).$$

$$\sum \frac{a}{bc(b+c)} = \frac{p^2-3r^2-4Rr}{2Rr(p^2+r^2+2Rr)}.$$

$$\sum \frac{a}{b+c} = \frac{2(p^2-r^2-Rr)}{p^2+r^2+2Rr}. \quad \sum \frac{b+c}{a} = \frac{p^2+r^2-2Rr}{2Rr}.$$

$$\sum \left(\frac{a}{b+c}\right)^2 = \frac{2[p^4 - p^2(4Rr+6r^2) + r^2(6R^2+4Rr+r^2)]}{(p^2+r^2+2Rr)^2}.$$

$$\sum a^2(a+b)^2(a+c)^2 = 8p^2[p^4 - p^2(4Rr+6r^2) + r^2(6R^2+4Rr+r^2)].$$

$$\sum a(b^2+c^2+a^2)(p-b)(p-c) = 8r^2 p [p^2 - R(4R+r)].$$

$$\sum \frac{a^2}{bc(b+c)} = \frac{p^4 - 6p^2(r^2 + Rr) + r^2(8R^2 + 6Rr + r^2)}{2Rrp(p^2 + r^2 + 2Rr)}.$$

$$\sum a^3(a+b)(a+c) = 4p [p^4 - 6p^2(r^2 + Rr) + r^2(8R^2 + 6Rr + r^2)].$$

$$\sum a(p-a) = 2r(4R+r). \quad \sum a^2(p-a) = 4rp(R+r).$$

$$\sum a^3(p-a) = 2 [p^2(2R+3r) - r(4R+r)^2].$$

$$\sum a^4(p-a) = 4rp [p^2(R+2r) - r(12R^2 + 11Rr + 2r^2)].$$

$$\sum a^5(p-a) = p^6 - p^4(8Rr + 23r^2) + p^2(80R^2r^2 + 60Rr^3 + 37r^4) - r^3(4R+r)^3.$$

$$\sum a(b+c) = 2(p^2 + r^2 + 4Rr). \quad \sum a^2(b+c) = 2p(p^2 + r^2 - 2Rr).$$

$$\sum a^3(b+c) = 2 [p^4 - 4p^2Rr - r^2(4R+r)^2].$$

$$\sum a^4(b+c) = 2p [p^4 - p^2(6Rr + 2r^2) - r^2(8R^2 + 14Rr + 3r^2)].$$

$$\sum a^5(b+c) = 3p^6 - p^4(28Rr + 43r^2) + p^2(160R^2r^2 + 120Rr^3 + 17r^4) - r^3(4R+r)^3.$$

$$\sum \frac{(b+c)^2}{a} = \frac{p(p^2 + r^2 - 6Rr)}{Rr}. \quad \sum bc(b+c)^2 = 4p^2(p^2 + r^2 - 6Rr).$$

$$\sum \frac{(b+c)^3}{a} = \frac{2 [p^4 + p^2(r^2 - 9Rr) + Rr(r^2 + 4Rr)]}{Rr}.$$

$$\sum bc(b+c)^3 = 8p [p^4 + p^2(r^2 - 9Rr) + Rr(r^2 + 4Rr)].$$

$$\sum \frac{(b+c)^2}{bc} = \frac{p^2 + r^2 + 10Rr}{2Rr}. \quad \sum \frac{(b+c)^3}{bc} = \frac{p^4 + 20p^2Rr - r^2(4R+r)^2}{2Rrp}.$$

$$\sum a(b+c)^3 = 2 [p^4 + 20p^2Rr - r^2(4R+r)^2].$$

$$\sum (b+c)^2 = 2(3p^2 - r^2 - 4Rr).$$

$$\sum (b+c)^2(p-b)(p-c) = 2(3p^2 - r^2 - 4Rr).$$

$$\sum \frac{b^2c^2}{(p-b)(p-c)} = \frac{p^4 + p^2(2r^2 - 12Rr) + r^3(4R+r)}{r^2}.$$

$$\sum b^2c^2(p-a) = p [p^4 + p^2(2r^2 - 12Rr) + r^3(4R+r)].$$

Respectiv pentru omenii și căm...

$$\sum \frac{a}{(p-b)(p-c)} = \frac{2(4R+r)}{rp}$$

$$\sum \frac{a^2}{(p-b)(p-c)} = \frac{4(R+r)}{r}$$

$$\sum \frac{a^3}{(p-b)(p-c)} = \frac{2[p^2(2R+3r) - r(4R+r)^2]}{rp}$$

$$\sum \frac{a^4}{(p-b)(p-c)} = \frac{4[p^2(R+2r) - r(12R^2 + 11Rr + 2r^2)]}{r}$$

$$\sum \frac{a^5}{(p-b)(p-c)} = \frac{p^6 - p^4(8Rr + 23r^2) + p^2(80R^2r^2 + 60Rr^3 + 37r^4) - r^3(4R+r)^3}{pr^2}$$

$$\sum \frac{a}{(p-b)^2(p-c)^2} = \frac{2(2R-r)}{r^3p}$$

$$\sum \frac{(p-b)(p-c)}{a} = \frac{rp}{4R} \left[\left(\frac{4R+r}{p} \right)^2 + 1 \right]$$

$$\sum \frac{b^2+c^2-a^2}{b+c-a} = \frac{5p^2 - (4R+r)^2}{p}$$

$$\sum \frac{b^2+c^2-a^2}{bc} = \frac{2(R+r)}{R}$$

$$\sum \frac{1}{bc} = \frac{1}{2Rr}$$

$$\sum \frac{b^2+c^2}{bc} = \frac{p^2+r^2-2Rr}{2Rr}$$

$$\sum \frac{(b+c)^2}{p-a} = \frac{4p(R+2r)}{r}$$

$$\sum \frac{(b+c)^2}{bc} = \frac{p^2+r^2+10Rr}{2Rr}$$

$$\sum \frac{b^2+c^2-a^2}{a} = \frac{p^4 - 8p^2Rr - r^2(4R+r)^2}{2Rrp}$$

$$\sum \frac{b^2+c^2-a^2}{a} = \frac{5p^2 - r^2(4R+r)^2}{p}$$

$$\sum \frac{(p-b)(p-c)}{b+c} = \frac{p^2(8Rr-3r^2) + r^2(4R+r)^2}{2p(p^2+r^2+2Rr)}$$

$$\sum \frac{b+c}{p-a} = 4 \left(1 + \frac{R}{r} \right)$$

$$\sum \frac{p-a}{b+c} = \frac{p^2+5r^2+8Rr}{2(p^2+r^2+2Rr)}$$

$$\sum \frac{bc}{b+c} \cos^2 \frac{A}{2} = \frac{p(p^2+5r^2+8Rr)}{2(p^2+r^2+2Rr)}$$

$$\sum a^2(p-a)^2 = 2r^2[(4R+r)^2 - p^2]$$

$$\sum bc(b+c) = 2p(p^2+r^2-2Rr)$$

$$\sum \frac{1}{a} = \frac{p^2+r^2+4Rr}{4Rrp}$$

$$\sum \frac{1}{a^2} = \frac{p^4 + p^2(2r^2 - 8Rr) + r^2(4R+r)^2}{16R^2r^2p^2}$$

$$\sum (b+c)^2 = 2(3p^2 - r^2 - 4Rr).$$

$$\sum \frac{1}{b+c} = \frac{5p^2 + r^2 + 4Rr}{2p(p^2 + r^2 + 2Rr)}.$$

$$\sum \frac{bc}{b+c} = \frac{p^4 + p^2(16Rr + 2r^2) + r^2(4R+r)^2}{2p(p^2 + r^2 + 2Rr)}.$$

$$\sum \frac{a(b+c)}{p-a} = \frac{4Rp}{r}.$$

$$\sum \frac{1}{p-a} = \frac{4R+r}{rp}.$$

$$\sum \frac{p}{p-a} = \frac{2(2R-r)}{r}.$$

$$\sum \frac{1}{a(p-a)} = \frac{p^2 + (4R+r)^2}{4Rrp^2}.$$

$$\sum \frac{a}{(p-b)(p-c)} = \frac{2(4R+r)}{rp}.$$

$$\sum \frac{(p-b)(p-c)}{p-a} = \frac{(4R+r)^2 - 2p^2}{p}.$$

$$\sum \frac{(p-b)(p-c)}{bc} = \frac{2R-r}{2R}.$$

$$\sum \frac{(p-a)^2}{a} = \frac{p(p^2 + r^2 - 12Rr)}{4Rr}.$$

$$\sum \frac{p-a}{bc} = \frac{4R+r}{2Rp}.$$

$$\sum \frac{p-a}{a^2} = \frac{p^4 + p^2(2r^2 - 12Rr) + r^3(4R+r)}{16R^2r^2p}.$$

$$\sum a(p-a) = 2r(4R+r).$$

$$\sum a(p-a)^2 = 2rp(2R-r).$$

$$\sum \frac{a}{(p-a)^2} = \frac{4R(4R+r) - 2p^2}{r^2p}.$$

$$\sum \frac{a(p-b)(p-c)}{p-a} = 4R(4R+r) - 2p^2.$$

$$\sum \frac{1}{(p-a)^2} = \frac{(4R+r)^2 - 2p^2}{r^2p^2}.$$

$$\sum a^2(p-a)^2 = 2r^2[(4R+r)^2 - p^2].$$

$$\sum \frac{p-a}{b+c} = \frac{p^2 + 5r^2 + 8Rr}{2(p^2 + r^2 + 2Rr)}.$$

$$\sum \frac{2p-a}{2p+a} = \frac{15p^2 - r^2 - 10Rr}{9p^2 + r^2 + 6Rr}.$$

$$\sum (2p-a)(2p+b)(2p+c) = 2p(15p^2 - r^2 - 10Rr).$$

$$\prod (2p+a) = 2p(9p^2 + r^2 + 6Rr).$$

$$\sum \frac{p-a}{p+a} = \frac{4p^2 - r^2 - 16Rr}{4p^2 + r^2 + 8Rr}.$$

$$\sum \frac{p-a}{a} = \frac{p^2 + r^2 - 8Rr}{4Rr}.$$

$$\sum (p-a)^2 = p^2 - 2r^2 - 8Rr.$$

$$\sum (p-a)^3 = p(p^2 - 12Rr).$$

$$\sum \frac{p-a}{(p-b)(p-c)} = \frac{p^2 - 2r^2 - 8Rr}{r^2p}.$$

$$\sum a^2(p-a) = 4rp(R+r).$$

$$\sum \frac{a^2}{p-a} = \frac{4p(R-r)}{r}.$$

$$\sum a^3(p-a) = 2r[p^2(2R+3r) - r(4R+r)^2].$$

$$\sum \frac{a^3}{p-a} = \frac{2p^2(2R-3r) + 2r^2(4R+r)}{r}.$$

$$\sum (p-a)^4 = p^4 - 16p^2Rr + 2r^2(4R+r)^2.$$

$$\sum \frac{1}{a^2(p-a)} = \frac{p^4 + p^2(2r^2 - 4Rr) + r(4R+r)^3}{16R^2r^2p^3}.$$

$$\sum a^2(p-b)(p-c) = 4rp^2(R-r).$$

$$\sum \frac{1}{a(p-a)^2} = \frac{p^2(r-8R) + (4R+r)^3}{4Rr^2p^3}.$$

$$\sum \frac{a(b+c)}{p-a} = \frac{4Rp}{r}.$$

$$\sum \frac{a(b+c)}{(p-b)(p-c)} = \frac{12R}{r}.$$

$$\sum a(b+c)(p-b)(p-c) = 4Rrp^2.$$

$$\sum (b+c)(p-b)(p-c) = 4rp(R+r).$$

$$\sum \frac{(p-b)(p-c)}{(a+b)(a+c)} = \frac{2r(R+r)}{p^2+r^2+2Rr}.$$

$$\sum a^2(a+b)(a+c) = 4p^2(p^2 - 3r^2 - 4Rr).$$

$$\sum bc(b+c)^2 = 4p^2(p^2 - 3r^2 - 4Rr).$$

$$\sum \frac{(b+c)^2}{a} = \frac{p(p^2+r^2-6Rr)}{Rr}.$$

$$\sum \frac{a}{bc(b+c)} = \frac{p^2 - 3r^2 - 4Rr}{2Rr(p^2+r^2+2Rr)}.$$

$$\sum b^2c^2(p-a) = p[p^4 + p^2(2r^2 - 12Rr) + r^3(4R+r)].$$

$$\sum a(p-a)^3 = 2r[2Rp^2 - r(4R+r)^2].$$

$$\sum \frac{(p-a)^2}{bc} = 1 - \frac{r}{2R}.$$

$$\sum \frac{(p-a)^3}{bc} = \frac{2Rp^2 - r(4R+r)^2}{2Rp}.$$

$$\sum a^2(p-b)^2(p-c)^2 = 2r^2p^2(8R^2+r^2-p^2).$$

$$\sum a^2(p-a)^3 = 4r^2p(4R^2 - 3Rr - r^2).$$

$$\sum \frac{1}{(p-b)^2(p-c)^2} = \frac{p^2 - 2r^2 - 8Rr}{r^4p^2}.$$

$$\sum \frac{1}{a(p-b)^2(p-c)^2} = \frac{p^2+r^2-12Rr}{4Rr^5p}.$$

$$\sum \frac{a^2}{(p-a)^2} = \frac{2(8R^2+r^2-p^2)}{r^2}.$$

$$\sum \frac{(p-a)^2}{b^2c^2} = \frac{(4R+r)^2 - p^2}{8R^2p^2}.$$

$$\sum a^3(p-b)(p-c) = 2rp[p^2(2R-3r) + r^2(4R+r)].$$

$$\sum b^2c^2(p-b)(p-c) = r^2[p^2(p^2 + 2r^2 - 4Rr) + r(4R+r)^3].$$

$$\sum \frac{bc}{a(p-a)} = \frac{p^2(p^2 + 2r^2 - 4Rr) + r(4R+r)^3}{4p^2Rr}.$$

$$\sum \frac{1}{a(p-b)(p-c)} = \frac{p^2 + r^2 - 8Rr}{4Rr^3p}. \quad \sum \frac{bc}{p-a} = \frac{p^2 + (4R+r)^2}{p}.$$

$$\sum (p-b)(p-c) = r(4R+r). \quad \sum \frac{bc}{(p-b)(p-c)} = \frac{p^2 + r^2 - 8Rr}{r^2}.$$

$$\sum \frac{1}{bc(p-b)(p-c)} = \frac{4R+r}{2Rr^2p^2}. \quad \sum \frac{1}{(p-b)(p-c)} = \frac{1}{r^2}.$$

$$\sum bc(p-a) = p(p^2 + r^2 - 8Rr). \quad \sum \frac{a^2}{(p-b)(p-c)} = 4\left(1 + \frac{R}{r}\right).$$

$$\sum a(b+c)(p-a) = 12Rrp.$$

$$\sum \frac{1}{a^2(p-b)(p-c)} = \frac{p^4 + p^2(2r^2 - 12Rr) + r^3(4R+r)}{16p^2R^2r^4}.$$

$$\sum (p-b)^2(p-c)^2 = r^2[(4R+r)^2 - 2p^2].$$

$$\sum a(p-b)^2(p-c)^2 = 2r^2p(8R^2 + 2Rr - p^2).$$

$$\sum \frac{a^3}{(p-b)(p-c)} = \frac{p^2(2R+3r) - r(4R+r)}{rp}.$$

$$\sum bc(p-b)(p-c) = r^2[p^2 + (4R+r)^2].$$

$$\sum bc(p-a)^2 = p^2(p^2 + r^2 - 12Rr). \quad \sum \frac{1}{bc(p-a)} = \frac{2R-r}{2Rr^2p}.$$

$$\sum \frac{(p-a)^2}{bc} = \frac{2R-r}{2R}. \quad \sum \frac{p-a}{bc} = \frac{4R+r}{2Rp}.$$

$$\sum \frac{(p-b)(p-c)}{a(p-a)} = \frac{p^2(r-8R) + (4R+r)^3}{4Rp^2}.$$

$$\sum \frac{bc}{p-a} = 1 + \left(\frac{4R+r}{p}\right)^2. \quad \sum a\left(\frac{1}{b} + \frac{1}{c}\right) = \frac{p^2 + r^2 - 2Rr}{2Rr}.$$

$$\sum a(p-a)^4 = 2S[2Rp^2 + r(r^2 - 2Rr - 24R^2)].$$

$$\sum \frac{(p-a)^4}{bc} = \frac{2Rp^2 + r(r^2 - 2Rr - 24R^2)}{2R}$$

$$\sum a^4(p-a) = 4S[p^2(R+2r) - r(2r^2 + 11Rr + 12R^2)]$$

$$\sum \frac{a^3(p-a)}{bc} = \frac{p^2(R+2r) - r(2r^2 + 11Rr + 12R^2)}{R}$$

$$\sum a(p-a)^3 = 4p^2Rr - 2r^2(4R+r)^2 \quad \sum \frac{(p-a)^3}{bc} = \frac{2p^2R - r(4R+r)^2}{2Rp}$$

$$\sum \frac{(b+c)(p-a)}{bc} = 3 \quad \sum \frac{p-a}{b^2c^2} = \frac{R+r}{4R^2S}$$

$$\sum \frac{p-a}{bc(b^2+c^2)} = \frac{p^6 + p^4(8Rr - 9r^2) + p^2r^2(112R^2 + 116Rr + 19r^2) - 3r^3(4R+r)^3}{8Rrp[p^6 + p^4(r^2 - 12Rr) + p^2r^2(40R^2 + 8Rr - r^2) - r^3(4R+r)^3]}$$

$$\begin{aligned} \sum a(p-a)(a^2+b^2)(a^2+c^2) &= \\ &= p^6 + p^4(8Rr - 9r^2) + p^2r^2(112R^2 + 116Rr + 19r^2) - 3r^3(4R+r)^3 \end{aligned}$$

$$\sum a^2(2b^2 + 2c^2 - a^2) = 2[p^4 + p^2(10r^2 - 8Rr) + r^2(4R+r)^2]$$

$$\prod(2b^2 + 2c^2 - a^2) = 4[p^6 + p^4 - p^2(60R^2r^2 + 120Rr^3 + 33r^4) - r^3(4R+r)^3]$$

$$\sum a^3(a+b)^2(a+c)^2 = 8p^3[p^4 - p^2(6Rr + 10r^2) + r^2(20R^2 + 18Rr + 5r^2)]$$

$$\sum \frac{a^3}{(b+c)^2} = \frac{2p[p^4 - p^2(6Rr + 10r^2) + r^2(20R^2 + 18Rr + 5r^2)]}{(p^2 + r^2 + 2Rr)^2}$$

$$\sum \frac{(p-b)(p-c)}{a^2} = \frac{p^4 + p^2(2r^2 - 4Rr) + r(4R+r)^3}{16R^2p^2}$$

$$\prod(b+c) = 2p(p^2 + r^2 + 2Rr)$$

$$\sum bc(a+b)(a+c) = p^4 + p^2(16Rr + 2r^2) + r^2(4R+r)^2$$

$$\sum \frac{1}{a(b+c)} = \frac{p^4 + p^2(16Rr + 2r^2) + r^2(4R+r)^2}{8Rrp^2(p^2 + r^2 + 2Rr)}$$

$$\sum (b+c)^2(a+b)(a+c) = 8p^2(p^2 + r^2 + 2Rr)$$

$$\sum \frac{b(p-b) + c(p-c)}{a(p-a)} = \frac{p^2(r-2R) + (4R+r)^3}{2p^2R}$$

$$\sum bc(p-b)(p-c)[b(p-b) + c(p-c)] = 2r^3[p^2(r-2R) + (4R+r)^3]$$